

Chapter 7, Section 4: Cables



Cables

discrete, concentrated loads



Cables carry/support loads
- always do so in tension

Assume:

- Weight is negligible — loads \gg cable's weight \Rightarrow cable will have straight (uncurved) sections
- Perfectly flexible — exhibits no resistance to bending ($M(x) = 0$)
- Inextensible — No change in the total length of the cable before/during/after loading

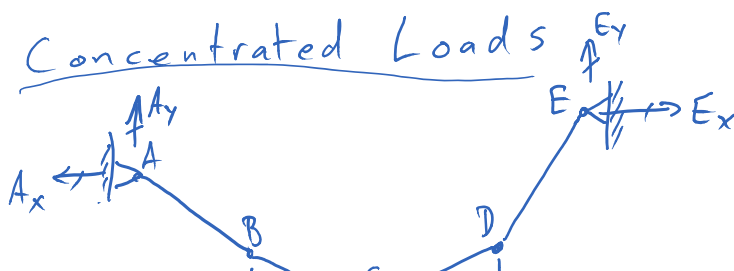
(in truth, there is "some" elongation, but insignificant compared to total length of the cable)

$$e \ll L$$

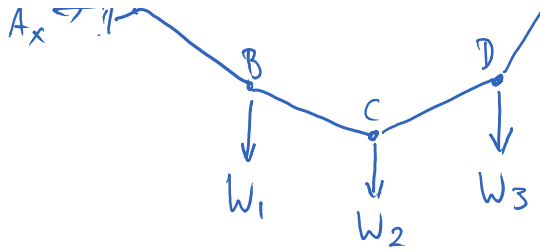
\uparrow elongation \uparrow cable length

In general, 3 cable problems

1. Concentrated Loads



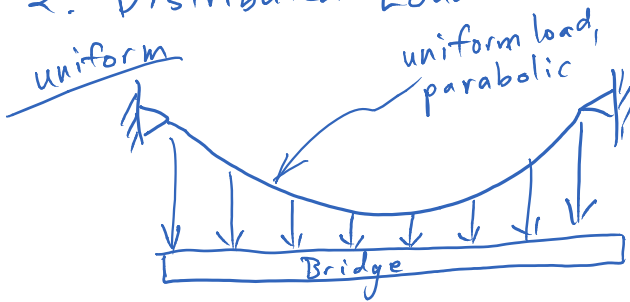
Straight segments between



segments
between
loads

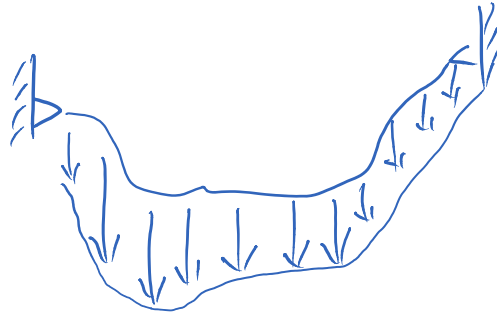
2. Distributed Loads

uniform



If distributed load is uniform, the cable curve $y(x)$ will be parabolic

non-uniform

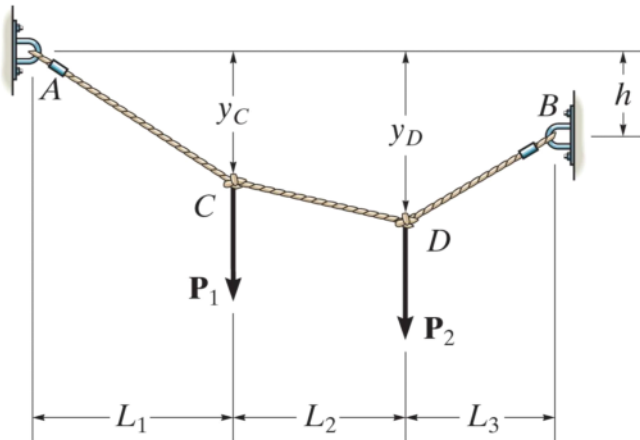


Cable subjected to its own weight alone



not parabolic
"catenary curve"

Cable subjected to concentrated loads



Givens (knowns)

$$h, P_1, P_2, L_1, L_2, L_3$$

Unknowns: T_{AC}, T_{CD}, T_{DB}

tension in each segment

$$A_x, A_y, B_x, B_y$$

sag at C & D

$$y_C \text{ \& } y_D$$

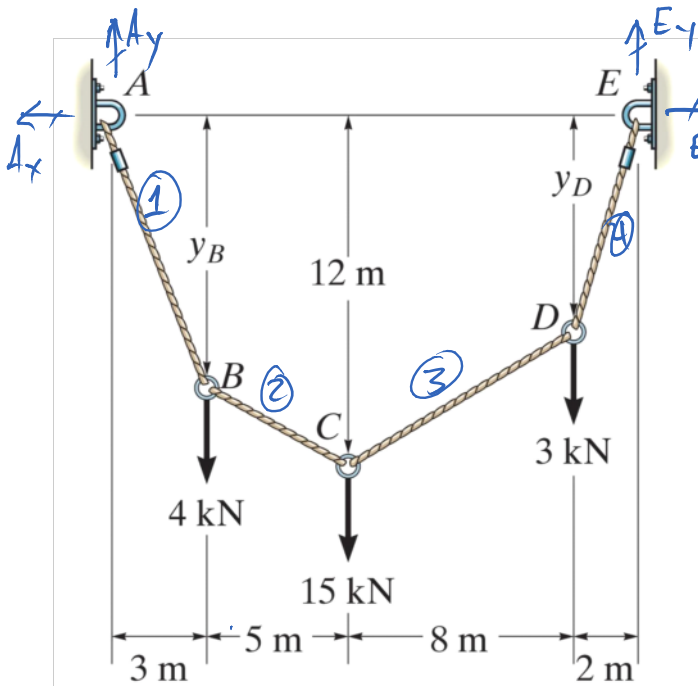
9 unknowns

Take $\sum F_x = 0$
 $\sum F_y = 0$ at each joint

$$A, B, C, D \Rightarrow 8 \text{ equations}$$

If we know initial cable length,
 we can use pythagorean theorem
 to get a 9th equation. \Rightarrow

problem will
 be hard to
 solve



Determine the tension in each segment of the cable shown.

If one sag is given, things become easier

unknowns

$$\left. \begin{matrix} A_x & A_y & E_x & E_y \\ T_1 & T_2 & T_3 & T_4 \\ y_B & y_D \end{matrix} \right\} (10)$$

$$\sum F_x = 0 \Rightarrow A_x = E_x$$

$$\sum F_y = 0 \Rightarrow A_y + E_y = 4\text{kN} + 15\text{kN} + 3\text{kN}$$

$$A_y + E_y = 22\text{kN}$$

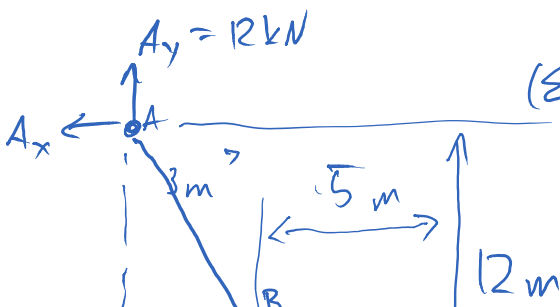
$$(\sum M)_E = 0$$

$$\Rightarrow (3\text{kN})(2\text{m}) + (15\text{kN})(10\text{m}) + (4\text{kN})(15\text{m}) - A_y(18\text{m}) = 0$$

$$\Rightarrow A_y = 12\text{kN}$$

$$E_y = 10\text{kN}$$

Cut segment 2:

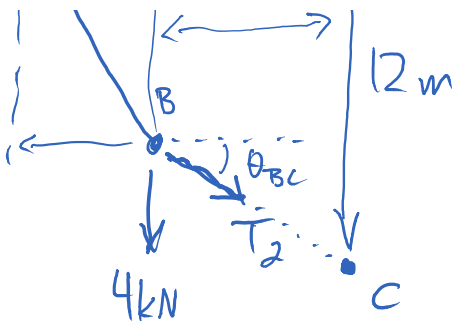


$$(\sum M)_C = 0$$

$$(12\text{m})A_x - (8\text{m})A_y + (5\text{m})(4\text{kN}) = 0$$

$$A_x = 6.33\text{kN}$$

$$E_x = 6.33\text{kN}$$



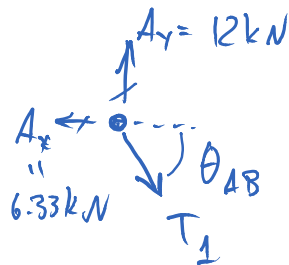
$$H_x = 6.33 \text{ kN} \quad | \quad E_x = 6.33 \text{ kN}$$

$$\begin{cases} \sum F_x = 0 \Rightarrow T_2 \cdot \cos \theta_{BC} = 6.33 \text{ kN} \\ \sum F_y = 0 \Rightarrow 12 \text{ kN} - 4 \text{ kN} - T_2 \cdot \sin \theta_{BC} = 0 \end{cases} \left. \begin{array}{l} 2 \text{ eqns} \\ 2 \text{ unknowns} \end{array} \right\}$$

Solve: $T_2 = 10.2 \text{ kN}$
 $\theta_{BC} = 51.6^\circ$

Equilibrium of A, C, & E

Equil. @ A:

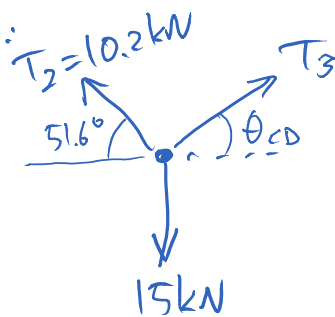


$$\sum F_x = 0 \Rightarrow T_1 \cdot \cos \theta_{AB} = 6.33 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow T_1 \cdot \sin \theta_{AB} = 12 \text{ kN}$$

Solve: $T_1 = 13.6 \text{ kN}$
 $\theta_{AB} = 62.2^\circ$

Equil. @ C:



$$\sum F_x = 0$$

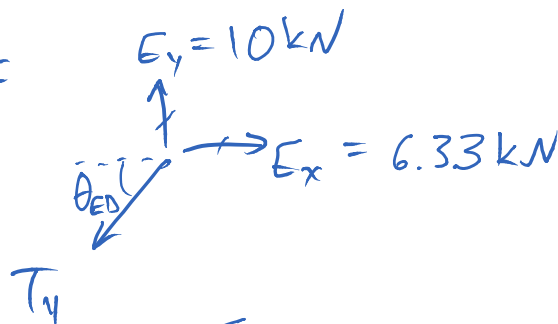
$$\Rightarrow T_2 \cdot \cos 51.6^\circ = T_3 \cdot \cos \theta_{CD}$$

$\underbrace{\hspace{10em}}_{6.33 \text{ kN}}$

$$\sum F_y = 0 \Rightarrow T_2 \sin \theta_{BC} + T_3 \sin \theta_{CD} = 15 \text{ kN}$$

Solve: $T_3 = 9.44 \text{ kN}$
 $\theta_{CD} = 47.9^\circ$

Equil. @ E:



$$\sum F_x = 0$$

$$\Rightarrow T_4 \cos \theta_{ED} = 6.33 \text{ kN}$$

$$\sum F_y = 0$$

$$\Rightarrow T_4 \sin \theta_{ED} = 10 \text{ kN}$$

$$\boxed{T_4 = 11.8 \text{ kN}}$$

$$\theta_{ED} = 57.7^\circ$$

we can now solve
for y_c & y_d

Summary

Rank	① $T_1 = 13.6 \text{ kN}$	$\theta_{AB} = 62.2^\circ$	①
	③ $T_2 = 10.2 \text{ kN}$	$\theta_{BC} = 51.6^\circ$	③
	④ $T_3 = 9.44 \text{ kN}$	$\theta_{CD} = 47.9^\circ$	④
	② $T_4 = 11.8 \text{ kN}$	$\theta_{DE} = 57.7^\circ$	②

Tension rank = slope rank

The horiz. component of
tension is same for all.